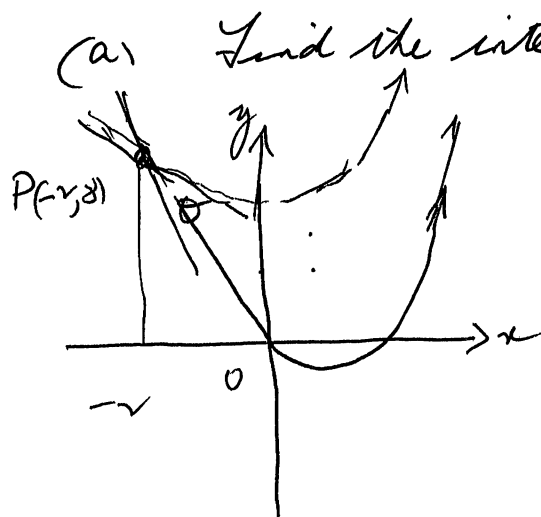


QUESTION 1. (X1)



$$2x = -4$$

$$x = -2.$$

$$\text{now } y = x^2 + 4 \quad \left| \quad y = x^2 - 2x \right.$$

$$y' = 2x \quad \left| \quad y' = 2x - 2 \right.$$

$$\therefore m_1 = -4 \quad \left| \quad m_2 = -6 \right.$$

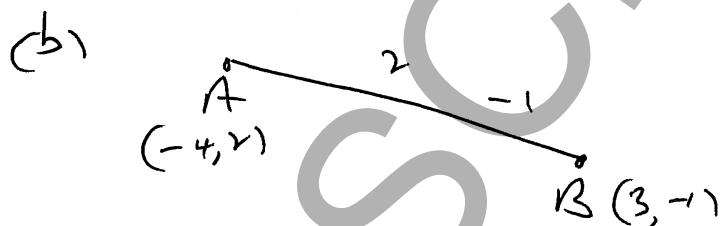
$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{-4 - (-6)}{1 + (-4)(-6)} \right|$$

$$= \left| \frac{-4 + 6}{1 + 24} \right|$$

$$= \frac{2}{25}$$

$$\therefore \theta = \tan^{-1} \frac{2}{25} = 4^\circ 34'$$



$$P = \left(\frac{2 \times 3 + (-1) \times (-4)}{2 + (-1)}, \frac{2 \times (-1) + (-1) \times 2}{2 + (-1)} \right)$$

$$= \boxed{(10, -4)}$$

(c) $y = \ln(\sin^2 x)$

$$y' = \frac{\frac{1}{\sqrt{1-x^2}}}{\sin^2 x}$$

$$= \boxed{\frac{1}{\sin^2 x \sqrt{1-x^2}}}$$

$$(d) \frac{x-1}{x+3} \geq -2$$

$$\frac{x-1}{x+3} + 2 \geq 0$$

$$\frac{x-1+2(x+3)}{x+3} \geq 0$$

$$\frac{x-1+2x+6}{x+3} \geq 0$$

$$\frac{3x+5}{x+3} \geq 0$$

$$\frac{(3x+5)}{(x+3)} \times (x+3)^2 \geq 0$$

$$(3x+5)(x+3) \geq 0$$

$$\begin{array}{c} \leftarrow 0 \qquad \qquad 0 \rightarrow \\ -3 \qquad \qquad -\frac{5}{3} \end{array}$$

$$\therefore \boxed{x < -3, x \geq -\frac{5}{3}}$$

$$(e)$$

$$\cos 2B = \cancel{2} 1 - 2 \sin^2 B$$

$$= 1 - 2 \times \left(\frac{1}{3}\right)^2$$

$$= 1 - \frac{2}{9}$$

$$= \frac{7}{9}$$

$$\therefore \cos 2B = \frac{7}{9} = \cos A$$

$$\boxed{A = 2B}$$

[NB Doing this question on a calculator is not a fail]

$$(f) u = t+1$$

$$du = dt$$

$$\therefore \int_0^1 \frac{t}{\sqrt{1+t}} dt = \int_{1/2}^2 \frac{u-1}{\sqrt{u}} du$$

$$= \int (u^{1/2} - u^{-1/2}) du$$

$$= \left[\frac{2}{3} u^{3/2} - 2u^{1/2} \right]_{1/2}^2 = \frac{2}{3} \cdot 2^{3/2} - 2 \cdot 2^{1/2} - \left(\frac{2}{3} \cdot \left(\frac{1}{2}\right)^{3/2} - 2 \cdot \left(\frac{1}{2}\right)^{1/2} \right)$$

$$= \frac{4}{3} - \frac{2\sqrt{2}}{3}$$

$$\boxed{\frac{4-2\sqrt{2}}{3}}$$

Question 2

$$a) P(x) = ax^3 + bx^2 + 8x + 3$$

$$P(1) = 0$$

$$\therefore 0 = a + b - 8 + 3$$

$$\boxed{a + b = 5} \quad (1)$$

$$P(-2) = 15$$

$$15 = -8a + 4b^2 + 16 + 3$$

$$\boxed{8a - 4b = 4} \quad (2)$$

$$(1) \times 4$$

$$4a + 4b = 20 \quad (3)$$

$$(2) + (3)$$

$$12a = 24$$

$$\boxed{a = 2} \quad 1 \text{ mark}$$

sub into (1)

$$2 + b = 5$$

$$\boxed{b = 3} \quad 1 \text{ mark}$$

$$\therefore P(x) = 2x^3 + 3x^2 - 8x + 3$$

$$2x^2 + 5x - 3$$

$$x-1 \overline{) 2x^3 + 3x^2 - 8x + 3}$$

$$2x^3 - 2x^2$$

$$5x^2 - 8x + 3$$

$$5x^2 + 5x$$

$$-3x + 3$$

$$-3x + 3$$

$$\underline{0}$$

$$2x^2 + 5x - 3$$

$$= \frac{(2x+6)(2x-1)}{2}$$

$$= \frac{2(x+3)(2x-1)}{2}$$

$$\therefore P(x) = (x-1)(x+3)(2x-1)$$

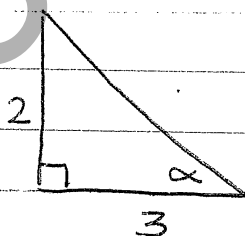
1 mark

$$b) i) 3\sin\theta + 2\cos\theta$$

$$= R\sin(\theta + \alpha)$$

$$R = \sqrt{3^2 + 2^2}$$

$$= \sqrt{13} \quad 1 \text{ mark}$$



$$\tan\alpha = \frac{2}{3}$$

$$\alpha = \tan^{-1} 2/3$$

$$= 33^\circ 41'$$

1 mark

$$ii) \sqrt{13} \sin(\theta + 33^\circ 41') = \frac{5}{2}$$

$$\sin(\theta + 33^\circ 41') = \frac{5}{2\sqrt{13}}$$

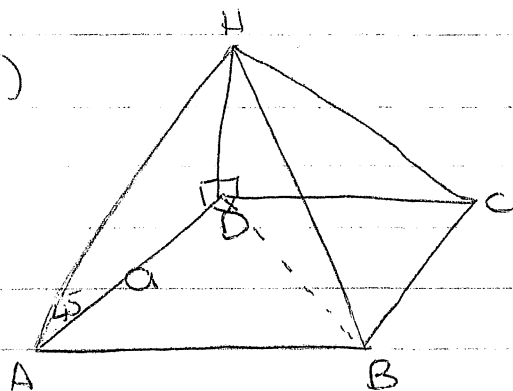
$$\theta + 33^\circ 41' = \sin^{-1} \frac{5}{2\sqrt{13}}$$

$$\theta = \sin^{-1} \frac{5}{2\sqrt{13}} - 33^\circ 41'$$

$$\theta = 10^\circ 13', 102^\circ 25'$$

1 mark each

c)



$$\tan \Theta = a/2a$$

$$\tan \Theta = 1/2$$

$$\Theta = 26^{\circ}33'54.18''$$

$$= 26^{\circ}34' \text{ (nearest min)}$$

1 mark

$$d) a_1 = a - \frac{f(a)}{f'(a)}$$

$$f(x) = 2x + \cos x$$

$$f'(x) = 2 - \sin x$$

$$a = -\pi/6$$

$$f(a) = \frac{-\pi}{3} + \frac{\sqrt{3}}{2}$$

$$f'(a) = 2 - -1/2$$

$$= 5/2$$

$$a_1 = -\pi/6 - \left[\frac{-2\pi + 3\sqrt{3}}{6} \right] \div \frac{5}{2}$$

1 mark

$$= \frac{-\pi}{6} - \left[\frac{-4\pi + 6\sqrt{3}}{30} \right]$$

$$= \frac{-5\pi}{30} + \frac{4\pi - 6\sqrt{3}}{30}$$

$$= \frac{-\pi - 6\sqrt{3}}{30} \text{ 1 mark}$$

$$i) \angle AHD = 45^{\circ}$$

$\therefore \Delta AHD$ is isosceles

$$\therefore HD = a$$

In Δ 's HCD + DBA

$$HD = a = DA$$

$$\angle HDC = 90^{\circ} = \angle DAB$$

(given & properties of a rectangle)

DC = AB (opposite sides of a rectangle)

$$\therefore \Delta HCD \equiv \Delta DBA \text{ (SAS)}$$

$$\therefore \angle DBA = 30^{\circ}$$

1 mark

In ΔOAB

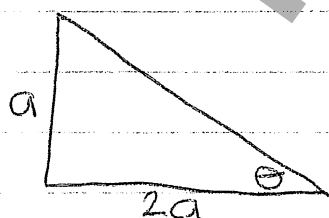
$$\sin 30 = \frac{a}{BD}$$

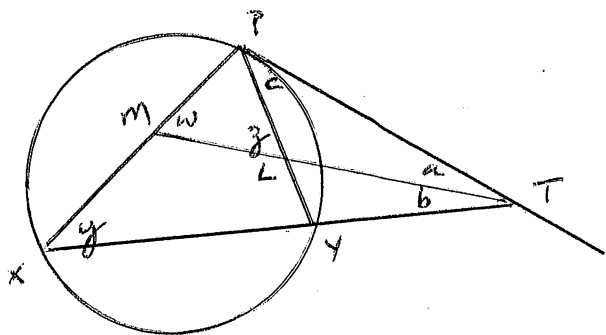
$$1/2 = a/BD$$

$$\therefore BD = 2a$$

1 mark

$$ii) \Delta HDB$$





$$= b + y$$

$$w = b + y \text{ (extension } < \text{ of } \Delta \text{ Mtx)}$$

$$\therefore z = w$$

$\therefore \triangle PLM$ is isosceles (base angles equal)

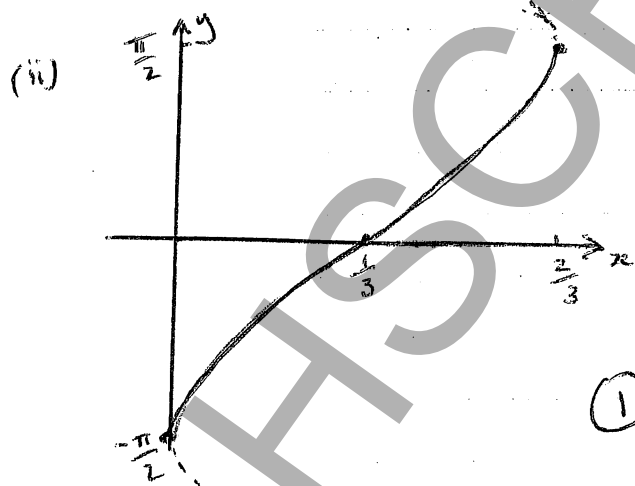
(b) (i) $f^{-1}(x) = \sin^{-1}(3x-1)$

Domaini: $-1 \leq 3x-1 \leq 1$

$$0 \leq j_c \leq 2$$

$$0 \leq x \leq \frac{2}{3}$$

Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



(iii) $x = \sin^{-1}(3y-1)$

$$\sin x = 3y - 1$$

$$3y = \sin x + 1$$

$$= \frac{1}{3} (\sin x + 1)$$

$$f(x) = \frac{1}{3} (\sin x + 1)$$

Domain: $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

Range : $0 \leq y \leq \frac{2}{3}$

(c) (i) $\frac{dT}{dt} = \frac{d}{dt} (T_0 + A e^{-kt})$

$$= A_2 - k_1 t \times -k$$

$$= (T - T_0) \times k$$

$$Q = k(T - T_0)$$

(ii) When $t=0$: $T = 85^\circ$

$$\therefore 85 = 25 + A$$

$$\therefore A = 60$$

$$\therefore T = 25 + 60 e^{-kt}$$

When $t=1$: $80 = 25 + 60 e^{-k}$

$$\therefore 55 = 60 e^{-k}$$

$$e^{-k} = \frac{55}{60}$$

$$K = -\ln\left(\frac{55}{60}\right)$$

When $t = 5$: $T = 25 + 60e^{-5k}$

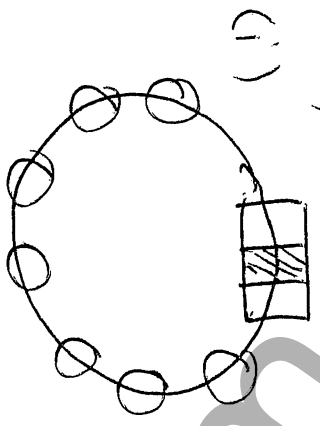
- 63 - 8336

$\approx 64^\circ$



Solution to Q(4)

(a)

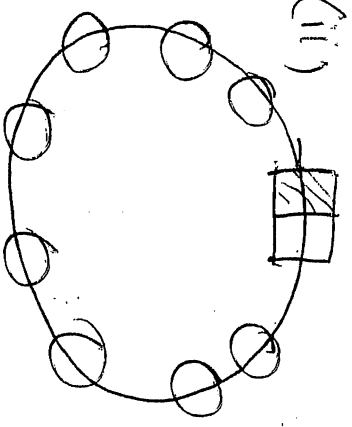


$$7! \times 1 \times 2!$$

the rest particular girl 2 boys

$$= \frac{2 \times 7!}{10080}$$

(ii)



Sitting together
 $= 8! \times 2!$
 = $\frac{9! - 8! \times 2!}{1}$

$$= 282240$$

(b) $y = x^{3/4} a$

$$\frac{dy}{dx} = \frac{x}{2a}$$

$$\frac{dy}{dx} \bigg|_{x=2at} = \frac{2at}{2a} = t$$

$$y - at^2 = k(x - 2at)$$

$$y = kx - at^2$$



$$R(0, -at^2)$$

[1]

If PQ passes through R, then coordinates of R satisfy equation of PQ

i.e. $-at^2 + apq = 0$ ✓

$$t^2 = pq$$

$$\frac{t}{p} = \frac{q}{t}$$

∴ p, t, q are terms of a geometric series

(c) $n = 2 \cos 3t$, $\dot{x} = -6 \sin 3t$

$$\dot{x} = \sqrt{9(2 \cos 3t)^2} = -9x$$

(i) $n^2 = 9$

(ii) $T = \frac{2\pi}{n} = \frac{2\pi}{3}$

(iii) $\omega = 3t = \frac{1}{2} \cdot \frac{2\pi}{3} = \frac{\pi}{3}$

$$\Rightarrow t = \frac{\pi}{9}, \frac{5\pi}{9}$$

$$x = -6 \sin \frac{\pi}{9}, -6 \sin \frac{5\pi}{9}$$

$$= \pm 3\sqrt{3} \text{ cm/sec}$$

∴ 15.196 cm/sec [2]

QUESTION 5.

(a) $\frac{\sin \theta}{\cos \theta} = 2 \sin \theta \cos \theta$

$$\sin \theta = 2 \sin \theta \cos^2 \theta.$$

$$2 \sin \theta (1 - \sin^2 \theta) - \sin \theta = 0.$$

$$2 \sin^3 \theta - \sin \theta = 0.$$

$$\sin \theta (2 \sin^2 \theta - 1) = 0.$$

$$\sin \theta = 0$$

$$\theta = \pi n \quad n \in \mathbb{Z}$$

$$\sin \theta = \pm \frac{1}{\sqrt{2}}$$

$$\theta = \pi n \pm \frac{\pi}{4} \quad n \in \mathbb{Z}$$

(b) (i) $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2}.$

(ii) $\alpha\beta\gamma = \frac{1}{2}.$

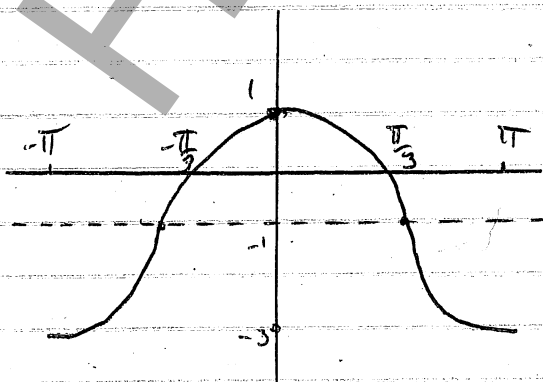
$$\frac{1}{\cos a} + \frac{1}{\cos b} + \frac{1}{\cos c} =$$

$$\frac{\cos a \cos b + \cos a \cos c + \cos b \cos c}{\cos a \cos b \cos c}.$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}}$$

$$= 1$$

(c) (i)



$$2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$x = \pm \frac{\pi}{3}.$$

(ii)

$$V = \pi \int_{-\pi/3}^{\pi/3} 4\cos^2 x - 4\cos x + 1 \, dx.$$

$$= \pi \int_{-\pi/3}^{\pi/3} 2\cos 2x + 2 - 4\cos x + 1 \, dx.$$

$$= \pi \left[\sin 2x - 4\sin x + 3x \right]_{-\pi/3}^{\pi/3}.$$

$$= 2\pi \left(\sin \frac{2\pi}{3} - 4\sin \frac{\pi}{3} + \pi \right).$$

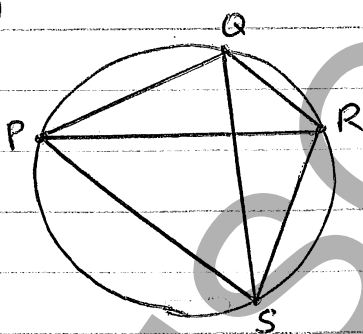
$$= 2\pi^2 - 3\pi\sqrt{3} \text{ units}^3.$$

Question 6

$$\begin{aligned} \text{a) i) } \frac{d(\ln \cos y)}{dy} &= \frac{-\sin y}{\cos y} \\ &= \underline{\underline{-\tan y}} \end{aligned}$$

$$\begin{aligned} \text{ii) } A &= \int_0^{\frac{\pi}{4}} x \, dy \\ &= \int_0^{\frac{\pi}{4}} \tan y \, dy \\ &= - \int_0^{\frac{\pi}{4}} -\tan y \, dy \\ &= - \left[\ln(\cos y) \right]_0^{\frac{\pi}{4}} \quad \text{using (i)} \\ &= - \left(\ln \left(\cos \frac{\pi}{4} \right) - \ln(\cos 0) \right) \\ &= - \ln \left(\frac{1}{\sqrt{2}} \right) + \ln 1 \\ &= - \ln(2)^{-\frac{1}{2}} \\ &= \frac{1}{2} \ln 2 \quad \text{units}^2 \end{aligned}$$

b)



In $\triangle PQR$

$$\frac{\sin \hat{PQR}}{PR} = \frac{\sin \hat{QRP}}{PQ}$$

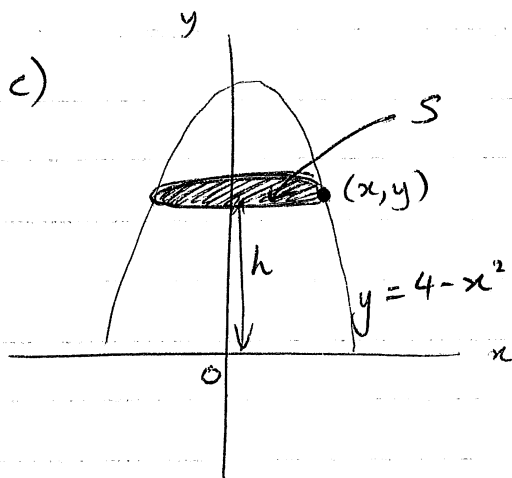
In $\triangle QPS$

$$\frac{\sin \hat{QPS}}{QS} = \frac{\sin \hat{PSQ}}{PQ}$$

$\hat{QRP} = \hat{PSQ}$ (angles in same segment (arc PQ))

$$\therefore \frac{\sin \hat{PQR}}{PR} = \frac{\sin \hat{QPS}}{QS}$$

$$\frac{\sin \hat{PQR}}{\sin \hat{QPS}} = \frac{PR}{QS}$$



i) $S = \pi r^2$
 $S = \pi x^2$

when $y = h$
 $h = 4 - x^2$
 $x^2 = 4 - h$

$\therefore S = \pi(4 - h)$

ii) $S = 4\pi - \pi h$

$\frac{dS}{dh} = -\pi$

$\frac{dS}{dt} = \frac{dS}{dh} \times \frac{dh}{dt}$
 $= -\pi \times \frac{10}{\pi(4-h)}$

$= -\frac{10}{4-h}$

when $h = 2$

$\frac{dS}{dt} = -\frac{10}{(4-2)}$

$= -5 \text{ cm}^2/\text{s}$

QUESTION 7

(a) $x = 20t \cos \alpha$
 $y = -5t^2 + 20t \sin \alpha$

(i) $\Rightarrow y = -5 \left(\frac{x}{20 \cos \alpha} \right)^2 + 20 \left(\frac{x}{20 \cos \alpha} \right) \sin \alpha$

$y = -\frac{1}{80} x^2 \sec^2 \alpha + x \tan \alpha$

ie $y = -\frac{1}{80} (\tan^2 \alpha + 1) x^2 + (\tan \alpha) x$

(ii) When $x = 20$, $y = h$

$\Rightarrow h = -\frac{1}{80} (\tan^2 \alpha + 1) 400 + 20 \tan \alpha$

ie $h = -5 \tan^2 \alpha + 20 \tan \alpha - 5$

$h = 20 \tan \alpha - 5(1 + \tan^2 \alpha)$

(iii)

$h = -5 \tan^2 \alpha + 20 \tan \alpha - 5$

Max. value of h occurs

when $\tan \alpha = \frac{-20}{2(-5)} = 2$

ie $\tan \alpha = 2$

Max height is

$-5(2)^2 + 20(2) - 5 = 15 \text{ metres}$

(b) $\frac{d}{dx} \left(\frac{1}{2} v^2 \right) = -9x + 5(x-2)^{-2}$

$\frac{1}{2} v^2 = -\frac{9x^2}{2} + \frac{5}{2-x} + C$

$\left. \begin{matrix} x=0 \\ v=0 \end{matrix} \right\} \Rightarrow C = -\frac{5}{2}$

$\therefore v^2 = -9x^2 + \frac{5}{2-x} - 5$

(b) $v^2 = -9x^2 + \frac{10}{2-x} - 5$

For motion to exist then

$v^2 \geq 0$

ie $-9x^2 + \frac{10}{2-x} - 5 \geq 0$

$-9x^2(2-x)^2 + 10(2-x) - 5(2-x)^2 \geq 0$

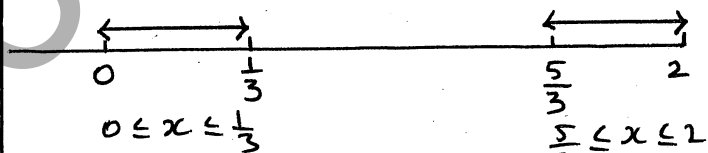
$(2-x) [-9x^2(2-x) + 10 - 5(2-x)] \geq 0$

ie $(2-x)(-18x^2 + 9x^3 + 5x) \geq 0$

ie $(2-x) \cdot x(9x^2 - 18x + 5) \geq 0$

$x(2-x)(3x-5)(3x-1) \geq 0$

SOLUTION



However since particle starts at zero and changes direction at $x = \frac{1}{3}$ it can never be outside the interval $0 \leq x \leq \frac{1}{3}$.

Note For $\frac{1}{3} < x < \frac{5}{3}$ $v^2 < 0$

* impossible to move in this interval and therefore cannot move in $\frac{5}{3} \leq x \leq 2$.

Ultimately moves in interval

$0 \leq x \leq \frac{1}{3}$

$$\begin{aligned}
 (7) \quad (c) \quad RHS &= \tan(\alpha + \beta) [1 - \tan \alpha \tan \beta] \\
 (i) \quad &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} [1 - \tan \alpha \tan \beta] \\
 &= \tan \alpha + \tan \beta \\
 &= LHS
 \end{aligned}$$

(ii) When $n=1$ $\tan \theta \cdot \tan 2\theta = \tan 2\theta \cot \theta - 2$

$$RHS = \frac{2 \tan \theta}{1 - \tan^2 \theta} \cdot \frac{1}{\tan \theta} - 2 = \frac{2 \tan^2 \theta}{1 - \tan^2 \theta} = \tan \theta \cdot \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$= \tan \theta \cdot \tan 2\theta = LHS$$

- Assume $\tan \theta \tan 2\theta + \dots + \tan k\theta \tan(k+1)\theta = \tan(k+1)\theta \cot \theta - (k+1)$

$$RTP \quad \tan \theta \tan 2\theta + \dots + \tan k\theta + \tan(k+1)\theta + \tan(k+1)\theta \tan(k+2)\theta = \tan(k+2)\theta \cot \theta - (k+2)$$

$$\text{Now } LHS = \tan(k+1)\theta \cdot \cot \theta - (k+1) + \tan(k+1)\theta \tan(k+2)\theta$$

$$= \cot \theta [\tan(k+1)\theta + \tan(k+1)\theta \cdot \tan(k+2)\theta \cdot \tan \theta] - (k+1)$$

$$= \cot \theta \left[\tan(k+1)\theta + \tan(k+2)\theta \left(1 - \frac{\tan(k+1)\theta + \tan \theta}{\tan(k+2)\theta} \right) \right] - (k+1)$$

$$= \cot \theta [\tan(k+1)\theta + \tan(k+2)\theta - \tan(k+1)\theta - \tan \theta] - (k+1) \quad \leftarrow \text{using (i)}$$

$$= \cot \theta [\tan(k+2)\theta - \tan \theta] - (k+1)$$

$$= \cot \theta [\tan(k+2)\theta] - 1 - (k+1)$$

$$= \cot \theta [\tan(k+2)\theta] - (k+2) = RHS$$