A: Yeivad Maths Extension 1 YEAR 12 TASK 2 2006
(HOD Copy) Term 1

TRINITY GRAMMAR SCHOOL

MATHEMATICS DEPARTMENT

YEAR 12 2006 ASSESSMENT TASK 2

MATHEMATICS

(EXTENSION 1)

Time Allowed – *1 hour*

Tuesday 28 February 2006

WEIGHTING 20% towards final result

Outcomes referred to: H1-H7, H9, PE2, PE3, PE5, PE7, HE2, HE3, HE7.

INSTRUCTIONS:

1. Attempt **ALL** questions
2. Show all necessary working.
3. Begin each question on a new page.
4. Write your name, your teacher's name and class on the top of each question
5. Mark values are shown beside each part.
6. Non-programmable silent Board of Studies approved calculators are permitted.

Students are permitted to bring one A4 page of double-sided hand written notes.

Name ______________________________

Question 1 **Start each question on a new page**

a) The rate of change of y with respect to time t is given by $\frac{dy}{dt} = (2t-1)^9$. If $y = \frac{1}{2}$ when $t = 1$, find y when $t = \frac{1}{2}$.

(2 marks)

b) The position of a particle moving along the x-axis is given by $x = t + e^{-t}$, where t is the time in seconds and x is measured in cm.

i) Show that the particle is at rest when $t = 0$.

ii) What is the velocity of the particle after 1 second? (Answer to 2 significant figures)

iii) What is the limiting velocity of the particle?

(6 marks)

c) The velocity of a particle is given by $v = 4\cos 2t \ \ m/s$. If the particle is 3m to the right of the origin after $\pi \ \ s$, find the exact:

i) Displacement after $\frac{\pi}{6}s$

ii) Acceleration after $\frac{\pi}{6}s$

(4 marks)

Question 2 **Start each question on a new page**

a) Find the acute angle between the lines $x+2y-5=0$ and $x-3y+3=0$.
(3 marks)

b) The point P(-3,8) divides the interval AB externally in the ratio k:1. If A is the point (6,-4) and B is the point (0,4), find the value of k.
(2 marks)

c) The functions $y=x^2-1$ and $y=3x-1$ intersect at the points P and Q.

i) Find the points P and Q.

ii) Find the acute angle between the two functions at their points of intersection.

(5 marks)

Question 3 **Start each question on a new page**

a) $(5x^3 + 7x^2 + 2x - 3) \div (x - 2) =$
(3 marks)

b) Use the Remainder Theorem to find the remainder when $P(x)$ is divided by $A(x)$ given $P(x) = 6x^4 - x^3 + x^2 - 5$ and $A(x) = x - 3$.
(2 marks)

c) Use the Factor Theorem to factorise $P(x) = 18 + 27x + x^2 - 4x^3$.
(3 marks)

d) Sketch $y = x(x-1)(x-2)(x-3)$ showing the intercepts.
(2 marks)

e) Find the values of m and n if $(x-2)$ is a factor of $P(x) = 6x^3 - 5mx^2 + 7nx + 10$ and there is a reminder of 10 when $P(x)$ is divided by $(x-3)$.
(3 marks)

Question 4 **Start each question on a new page**

a) One card is selected at random from a pack of 52 cards. What is the probability that the card is:

i) A diamond
ii) Not a picture card
iii) Either a red or an ace

(3 marks)

b) A class of 30 pupils contains 19 pupils who watch cartoons on T.V., 12 who watch 'soap operas' and 5 who watch both cartoons and soap operas. If one of the class is chosen at random, find the probability that this pupil watches:

i) Either cartoons or soap operas, but not both.
ii) Neither cartoons nor soap operas.

(2 marks)

c) Steven has 5 tickets in a raffle in which there are 2 prizes and 50 tickets sold. Find the probability that he wins:

i) First prize
ii) Both prizes
iii) Only the second prize
iv) A prize

(4 marks)

d) The probability that a particular type of shrub will flower in the first year after planting is $\frac{2}{5}$. How many of these shrubs need to be planted in order to be more than 95% certain of having at least one flowering shrub in the first year?

(4 marks)

Question 5 Start each question on a new page

a) Prove by mathematical induction, that $4^n - 1$ is divisible by 3 for all $n \geq 1$.
(4 marks)

b) Prove by mathematical induction $\sum_{r=1}^{n} 3(2^r) = 6(2^n - 1)$

(4 marks)

Question 6 **Start each question on a new page**

a) Solve:

i) $\frac{x^2}{(x+2)(x+3)} \leq 1$

ii) $\frac{4}{5-x} \geq 1$

(5 marks)

b) The polynomial $P(x) = x^3 + bx^2 + cx + d$ has roots 0, 3 and -3.

i) Find b, c, d.

ii) Without calculus, sketch the graph of $y = P(x)$.

(5 marks)

END OF TEST

A YARAD Maths Extension 1 Total ⇒ 66 2006
(HOD copy) YEAR 12 TASK 2 Term 1

Question 1 $\boxed{12}$

a) $\frac{dy}{dt} = (2t-1)^9$

$y = \frac{(2t-1)^{10}}{20} + c$

$y=\frac{1}{2}, t=1$ ∴ $\frac{1}{2} = \frac{(2(1)-1)^{10}}{20} + c$

$\frac{1}{2} = \frac{1}{20} + c$

$c = \frac{9}{20}$ — 1

∴ $y = \frac{(2t-1)^{10}}{20} + \frac{9}{20}$

$t=\frac{1}{2}$, $y = \frac{(2(\frac{1}{2})-1)^{10}}{20} + \frac{9}{20}$

$= \frac{9}{20}$ — 1

b) $x = t + e^{-t}$

i) show v=0 $v = 1 - e^{-t}$ — 1

$t=0$, $v = 1 - e^0$

$= 1 - 1$

$= 0$ — 1

∴ at rest

ii) $V = 1 - e^{-t}$

$t = 1$, $V = 1 - e^{-1}$

$= 1 - \frac{1}{e}$ — (1)

$= 0.63212\ldots$

≈ 0.63 cm/s (2 sig figs)] — (1)

iii) limiting velocity as $t \to \infty$ — (1) realise need $t \to \infty$

$\therefore V = 1 - \frac{1}{e^t}$

$t \to \infty$, $\frac{1}{e^t} \to 0$

$\therefore$ limiting velocity is 1 cm/s — (1)

c) $V = 4\cos 2t$

$x = 3, t = \pi$ i) $x = \frac{4\sin 2t}{2} + c$

$3 = 2\sin 2\pi + c$

$\therefore 3 = c$ — (1)

$x = 2\sin 2t + 3$

$t = \frac{\pi}{6}$, $x = 2\sin \frac{2\pi}{6} + 3$

$= 2 \cdot \frac{\sqrt{3}}{2} + 3$

$= (\sqrt{3} + 3)$ m — (1)

ii) $v = 4\cos 2t$

$a = -8\sin 2t$ — 1

$t = \frac{\pi}{6}$, $a = -8.\sin\frac{\pi}{3}$

$= -8.\frac{\sqrt{3}}{2}$

$a = -4\sqrt{3}$ m/s² — 1

2 a) $x + 2y - 5 = 0$ $\quad$ $x - 3y + 3 = 0$

$\boxed{10}$

$2y = -x + 5$ $\quad$ $3y = x + 3$

$y = -\frac{1}{2}x + \frac{5}{2}$ $\quad$ $y = \frac{1}{3}x + 1$

$m_1 = -\frac{1}{2}$ — 1 $\quad$ $m_1 = \frac{1}{3}$ — 1

$\tan\theta = \dfrac{-\frac{1}{2} - \frac{1}{3}}{1 + \frac{-1}{6}}$

$= -1$

$\theta = 135°$

$\therefore$ acute $\angle$ is $45°$ — 1

b) $-3 = \dfrac{k \times 0 + -1 \times 6}{k + -1}$ — 1

$-3 = \dfrac{-6}{k-1}$

$-3k + 3 = -6$

$-3k = -9$

$\boxed{k = 3}$ — 1

c) $y = x^2 - 1$ and $y = 3x - 1$

i) $x^2 - 1 = 3x - 1$

$x^2 - 3x = 0$

$x(x-3) = 0$

$\therefore x = 0, x = 3$

$y = 1, y = 8$

$\therefore P(0,1)$ ① and $Q(3,8)$ ①

ii) $\boxed{\begin{array}{l|l} y = x^2 - 1 & y = 3x - 1 \\ y' = 2x & y' = 3 \end{array}}$ — ① B. ~~A~~

At P:

$x = 0, y' = 0$

$m_1 = 0$

and $m_2 = 3$

$\therefore \tan\theta = \frac{0-3}{1+0}$

$= -3$

$\therefore \theta = 71°33'54.18''$

$\doteq 71°34'$ — ①

at Q:

$x = 3, y' = 6$

$\therefore m_1 = 6$

and $m_2 = 3$

$\tan\theta = \frac{6-3}{1+6(3)}$

$= \frac{3}{19}$

$\therefore \theta = 8°58'21.46''$

$\doteq 8°58'$. — ①

3) a)

$$\begin{array}{r} 5x^2 + 17x + 36 \\ x-2 \overline{) 5x^3 + 7x^2 + 2x - 3} \\ \underline{5x^3 - 10x^2} \\ 17x^2 + 2x \\ \underline{17x^2 - 34x} \\ 36x - 3 \\ \underline{36x - 72} \\ 69 \end{array}$$

$17x^2 + 2x$ — (1) for correct start to division

13

$\therefore = 5x^2 + 17x + 36 + \dfrac{69}{x-2}$

(1) (1)

b) Using remainder theorem

$P(3) = 6(3^4) - 3^3 + 3^2 - 5$ — (1)

$= 463$

$\therefore$ remainder is 463 — (1)

c) $P(x) = 18 + 27x + x^2 - 4x^3$

$P(-2) = 18 + 27(-2) + (-2)^2 - 4(-2^3)$

$= 0$

$\therefore (x+2)$ is a factor. — (1)

$$\begin{array}{r} -4x^2 + 9x + 9 \\ x+2 \overline{) -4x^3 + x^2 + 27x + 18} \\ \underline{-4x^3 - 8x^2} \\ 9x^2 + 27x \\ \underline{9x^2 + 18x} \\ 9x + 18 \\ \underline{9x + 18} \\ 0 \end{array}$$

— (1) for correct start to division.

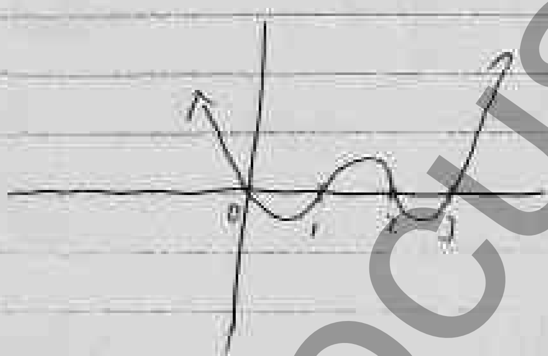
$\therefore\ 18 + 27x + x^2 - 4x^3 = (x+2)(-4x^2+9x+9)$

$= -(x+2)(4x^2-9x-9)$

$= -(x+2)(4x+3)(x-3)$ — 1

d) $y = x(x-1)(x-2)(x-3)$

$y=0,\ x=0, 1, 2, 3$ — 1 for all intercepts



— 1 for graph.

e) $P(x) = 6x^3 - 5mx^2 + 7nx + 10$

$P(2)=0$

$\therefore\ 6(2^3) - 5m(2^2) + 7n(2) + 10 = 0$

$58 - 20m + 14n = 0$

$\therefore\ \boxed{10m - 7n = 29}$

1

$P(3) = 10$

$\therefore\ 6(3^3) - 5m(3^2) + 7n(3) + 10 = 10$

$162 - 45m + 21n + 10 = 10$

$\boxed{15m - 7n = 54}$

1

solving: $5m = 25$

$\boxed{m = 5}$

$\therefore\ 10(5) - 7n = 29$

$-7n = -21$

$\boxed{n = 3}$

1 for both.

(Fractions don't need to be simplified...)

4) a) i) $\frac{13}{52} = \frac{1}{4}$ — 1

$\boxed{13}$

ii) $\frac{40}{52} = \frac{10}{13}$ — 1

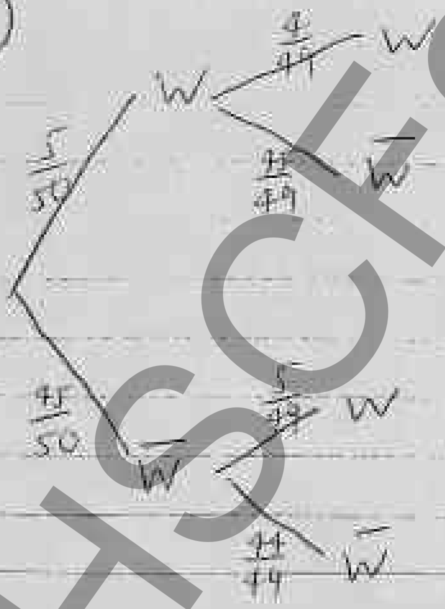
iii) $\frac{28}{52} = \frac{7}{13}$ — 1

b)

i) $\frac{21}{30} = \frac{7}{10}$ — 1

ii) $\frac{4}{30} = \frac{2}{15}$ — 1

c)



i) $P(W) = \frac{5}{50} = \frac{1}{10}$ — 1

ii) $P(WW) = \frac{5}{50} \times \frac{4}{49}$

$= \frac{2}{245}$ — 1

iii) $P(\overline{W}W) = \frac{45}{50} \times \frac{5}{49}$

$= \frac{9}{98}$ — 1

iv) $1 - (P\overline{W}\overline{W}) = 1 - \left(\frac{45}{50} \times \frac{44}{49}\right)$

$= \frac{47}{245}$ — 1

d) $P(F) = \frac{2}{5} = 0.4$

$P(\bar{F}) = \frac{3}{5} = 0.6$ — 1

in n years:

$P(\bar{F}) = 0.6^n$

$\therefore P(F) = 1 - 0.6^n$ — 1

need: $1 - 0.6^n \geq 0.95$ — 1

$-0.6^n > -0.05$

$0.6^n < 0.05$

$\ln 0.6^n < \ln 0.05$

$n \ln 0.6 < \ln 0.05$

$n > \frac{\ln 0.05}{\ln 0.06}$

> 5.86

$\therefore n = 6$ sharks. — 1

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5 a) $4^n - 1$ div. by 3

Step ① – show true for n=1 : $4^1 - 1 = 3$ — 1
which is div. by 3.

Step ② – assume true for $n=k$
ie $4^k - 1$ is div. by 3
ie $4^k - 1 = 3m$ (where m is an integer) — 1

prove true for $n=k+1$; $4^{k+1} - 1 = 4^{k+1} - 4 + 3$
$= 4(4^k - 1) + 3$
$= 4(3m) + 3$
$= 12m + 3$
$= 3(4m+1)$ — 1
which is div. by 3.

Hence if it is true for n=k, then it is true for n=k+1 and so on.
step ③ it is true for n=1 and so it is true when n=2 and so on.
Hence true for all $n \geq 1$. — 1

b) $$\sum_{r=1}^{n} 3(2^r) = 6(2^n - 1) \quad \therefore 6 + 12 + 24 + \dots + 3(2^n) = 6(2^n - 1)$$

① : show true for $n=1$: $LHS = 3(2^1) = 6$

$RHS = 6(2^1 - 1) = 6$

$LHS = RHS$ $\therefore$ true for $n=1$. — 1

② assume true for $n=k$ $\therefore 6 + 12 + 24 + \dots 3(2^k) = 6(2^k - 1)$

prove true for $n=k+1$

$\therefore 6 + 12 + 24 + \dots 3(2^k) + 3(2^{k+1}) = 6(2^{k+1} - 1)$ — 1

$LHS = 6(2^k - 1) + 3(2^{k+1})$

$= 6(2^k) - 6 + 3(2^k . 2^1)$

$= 6(2^k) - 6 + 6(2^k)$

$= 12(2^k) - 6$

$= 6(2^1 . 2^k - 1)$

$= 6(2^{k+1} - 1)$

$= RHS$. 1

③ Hence if true for $n=k$, then it is true for $n=k+1$ ~~and vice~~.
Since result is true for $n=1$, then true for $n=2$ and so on for $n \geq 1$. 1

$\boxed{10}$

(6) i) $\dfrac{x^2}{(x+2)(x+3)} \le 1$ $\qquad x \ne -2,\ x \ne -3$

$x^2(x+2)(x+3) \le [(x+2)(x+3)]^2$ — (1)

$[(x+2)(x+3)]^2 - x^2(x+2)(x+3) \ge 0$

$(x+2)(x+3)[(x+2)(x+3) - x^2] \ge 0$

$(x+2)(x+3)(x^2+5x+6-x^2) \ge 0$

$(x+2)(x+3)(5x+6) \ge 0$ — (1)

sol: $-3 < x < -2$ and $x \ge -1\frac{1}{5}$ — (1) $\qquad$ take off one if symbols not right.

ii) $\dfrac{4}{5-x} \ge 1$ $\qquad x \ne 5$

$4(5-x) \ge (5-x)^2$ — (1)

$4(5-x) - (5-x)^2 \ge 0$

$(5-x)[4-(5-x)] \ge 0$

$(5-x)(x-1) \ge 0$

$\therefore$ sol: $1 \le x < 5$ — (1) $\qquad$ take of one mark if symbols not right.

note $x \ne 5$.

b) $P(x) = x^3 + bx + cx + d$

i) Roots : $\alpha = 0$, $\beta = 3$, $\gamma = -3$

$\therefore \alpha + \beta + \gamma = -b$

$0 + 3 - 3 = -b$

$\therefore \underline{b = 0}$ — 1

$\alpha\beta + \alpha\gamma + \beta\gamma = c$

$0 + 0 + 3(-3) = c$

$\underline{c = -9}$ — 1

$\alpha\beta\gamma = -d$

$0 = -d$

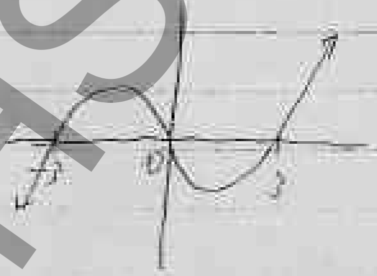
$\underline{d = 0}$ — 1

ii) $y = P(x)$

$y = x^3 - 9x$

$= x(x^2 - 9)$

$y = x(x-3)(x+3)$ — 1



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