SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2005

YEAR 12

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Working time – 2 Hours.
- Reading Time – 5 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for messy or badly arranged work
- Hand in your answer booklets in 4 sections. Section A (Questions 1 and 2), Section B (Questions 3 and 4), Section C (Questions 5 and 6) and Section D (Question 7)

Total Marks - 84

- Attempt questions 1 – 7
- All QUESTIONS are of equal value.

Examiner: *A. Fuller*

Total marks - 84
Attempt Questions 1 - 7
All questions are of equal value

Answer each SECTION in a SEPARATE writing booklet.

Section A

Marks

Question 1 (12 marks)

(a) Simplify $\dfrac{3^n}{3^{n+1}-3^n}$ **1**

(b) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 5x}{4x}$ **1**

(c) The remainder when $x^3 - 3x^2 + px - 14$ is divided by $x - 3$ is 1. Find the value of p. **2**

(d) Given that $\log_a 2 = x$, find $\log_a(2a)$ in terms of x. **2**

(e) Find the coordinates of the point P that divides the interval from A (-1,5) to B (6,-4) externally in the ratio 3 : 2 . **2**

(f) Find, to the nearest minute, the acute angle between The lines $3x + 2y - 5 = 0$ and $x - 5y + 7 = 0$. **2**

(g) Solve the inequality $\dfrac{2}{x} \leq 1$ **2**

Question 2 (12 marks)

(a) Differentiate with respect to x

(i) $y = \tan^3(5x+4)$ **2**

(ii) $y = \ln\left(\dfrac{2x+3}{3x+4}\right)$ **2**

(iii) $y = \cos\left(e^{1-5x}\right)$ **2**

(b) 30 girls, including Miss Australia, enter a Miss World Competition. The first six places are announced.

(i) How many different announcements are possible? **1**

(ii) How many different announcements are possible if Miss Australia is assured a place in the first six? **2**

(c) If $f(x) = \tan^{-1}(2x)$ evaluate:

(i) $f\left(\frac{1}{2}\right)$ **1**

(ii) $f'\left(\frac{1}{2}\right)$ **2**

End of Section

Section B (Use a SEPARATE writing booklet)

Marks

Question 3 (12 marks)

(a) (i) State the natural domain and the corresponding range of $y = 3\cos^{-1}(x-2)$ **2**

(ii) Hence, or otherwise sketch $y = 3\cos^{-1}(x-2)$ **1**

(b) Find $\int x\sqrt{16+x^2}\,dx$ using the substitution $u = 16 + x^2$ **2**

(c) Find the general solution of $\sin 2\theta = \sqrt{3}\cos 2\theta$ **2**

(d) The roots of the equation $4x^3 + 6x^2 + c = 0$, where c is a non-zero constant, are α, β, and $\alpha\beta$. **5**

(i) Show that $\alpha\beta \neq 0$.

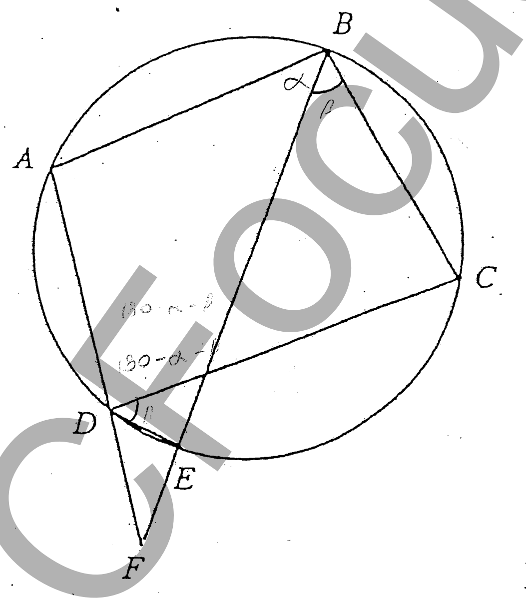
(ii) Show that $\alpha\beta + \alpha^2\beta + \alpha\beta^2 = 0$ and deduce the value of $\alpha + \beta$.

(iii) Show that $\alpha\beta = -\dfrac{1}{2}$.

Question 4 (12 marks)

(a) If $\tan\theta = 2$ and $0 < \theta < \frac{\pi}{2}$ evaluate $\sin\left(\theta + \frac{\pi}{4}\right)$. **3**

(b) In the diagram ABCD is a cyclic quadrilateral. The bisector of $\angle$ABC cuts the circle at E, and meets AD produced at F.

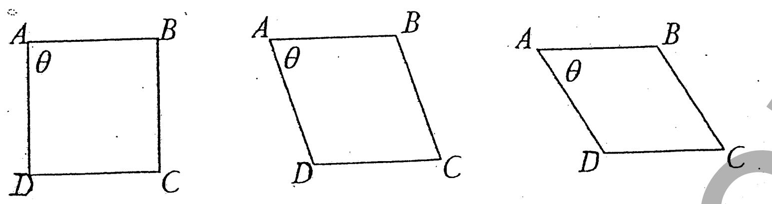


(i) Copy the diagram showing the above information

(ii) Give a reason why $\angle$CDE=$\angle$CBE **1**

(iii) Show that DE bisects $\angle$CDF **3**

(c)



A square ABCD of side 1 unit is gradually 'pushed over' to become a rhombus. The angle at A (θ) decreases at a constant rate of $0 \cdot 1$ radians per second.

(i) At what rate is the area of the rhombus ABCD decreasing when $\theta = \frac{\pi}{6}$? **2**

(ii) At what rate is the shorter diagonal of the rhombus ABCD decreasing when $\theta = \frac{\pi}{3}$? **3**

End of Section

Section C (Use a SEPARATE writing booklet)

Marks

Question 5 (12 marks)

(a) Two boys decide to settle an argument by taking turns to toss a die. The first person to throw a six wins.

(i) What is the probability that the first person wins on his second throw? **1**

(ii) What is the probability that the first person will win the argument? **2**

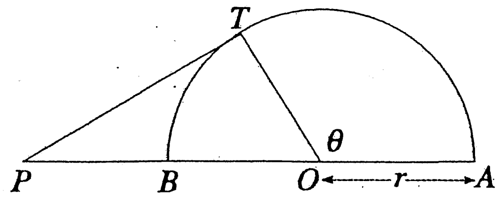
(b) $P(2at, at^2)$, $t > 0$ is a point on the parabola $x^2 = 4ay$. The normal to the parabola at P cuts the x axis at X and the y axis at Y.

(i) Show that the normal at P has equation $x + ty - 2at - at^3 = 0$ **2**

(ii) Find the co-ordinates of X and Y **1**

(iii) Find the value of t such that P is the midpoint of XY **2**

(c)



The point T lies on the circumference of a semicircle, radius r and diameter AB, as shown. The point P lies on AB produced and PT is the tangent at T. **4**

The arc AT subtends an angle of θ at the centre, O, and the area of ΔOPT is equal to that of the sector AOT.

(i) Show that $\theta + \tan\theta = 0$.

(ii) Taking 2 as an approximation to θ, use Newton's method once to find a better approximation to two decimal places.

Question 6 (12 marks)

(a) A particle is oscillating in simple harmonic motion such that its displacement x metres from a given origin O satisfies the equation $\frac{d^2x}{dt^2} = -4x$ where t is the time in seconds

(i) Show that $x = \alpha \cos(2t + \beta)$ is a possible equation of motion for this particle, where α and β are constants **2**

(ii) The particle is observed initially to have a velocity of 2 metres per second and a displacement from the origin of 4 metres. Find the amplitude of the oscillation. **2**

(iii) Determine the maximum velocity of the particle **2**

(b) Prove by Mathematical Induction that **3**

$$\sum_{r=1}^{n} r^3 = 1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \frac{1}{4}n^2(n+1)^2$$

(c) Consider the function $f(x) = \frac{x}{\sqrt{1-x^2}}$

(i) Find the domain of $f(x)$ **1**

(ii) Find $f^{-1}(x)$, the inverse function of $f(x)$ **2**

End of Section

Section D (Use a SEPARATE writing booklet)

Marks

Question 7 (12 marks)

(a) A projectile fired with velocity V and at an angle of 45° to the horizontal, just clears the tops of two vertical posts of height $8a^2$, and the posts are $12a^2$ apart. There is no air resistance, and the acceleration due to gravity is g.

(i) If the projectile is at a point $P(x, y)$ at time t, Derive expressions for x and y in terms of t. **2**

(ii) Hence, show that the equation of the path of the projectile is $y = x - \dfrac{gx^2}{V^2}$ **2**

(iii) Using the information in (ii) show that the range of the projectile is $\dfrac{V^2}{g}$ **2**

(iv) If the first post is b units from the origin, show that **2**

(α) $\dfrac{V^2}{g} = 2b + 12a^2$

(β) $8a^2 = b - \dfrac{gb^2}{V^2}$

(v) Hence or otherwise prove that $V = 6a\sqrt{g}$ **4**

End of paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right)\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$