



**SYDNEY BOYS HIGH  
SCHOOL**  
MOORE PARK, SURRY HILLS

**2005**

**YEAR 12**

**TRIAL HIGHER SCHOOL  
CERTIFICATE EXAMINATION**

# Mathematics Extension 1

## General Instructions

- Working time – 2 Hours.
- Reading Time – 5 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for messy or badly arranged work
- Hand in your answer booklets in 4 sections. Section A (Questions 1 and 2), Section B (Questions 3 and 4), Section C (Questions 5 and 6) and Section D (Question 7)

## Total Marks - 84

- Attempt questions 1 – 7
- All QUESTIONS are of equal value.

Examiner: *A. Fuller*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate

Total marks - 84

Attempt Questions 1 - 7

All questions are of equal value

Answer each SECTION in a SEPARATE writing booklet.

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Section A

Marks

Question 1 (12 marks)

- (a) Simplify  $\frac{3^n}{3^{n+1} - 3^n}$  1
- (b) Evaluate  $\lim_{x \rightarrow 0} \frac{\sin 5x}{4x}$  1
- (c) The remainder when  $x^3 - 3x^2 + px - 14$  is divided by  $x - 3$  is 1. Find the value of  $p$ . 2
- (d) Given that  $\log_a 2 = x$ , find  $\log_a(2a)$  in terms of  $x$ . 2
- (e) Find the coordinates of the point  $P$  that divides the interval from  $A(-1, 5)$  to  $B(6, -4)$  externally in the ratio  $3 : 2$ . 2
- (f) Find, to the nearest minute, the acute angle between the lines  $3x + 2y - 5 = 0$  and  $x - 5y + 7 = 0$ . 2
- (g) Solve the inequality  $\frac{2}{x} \leq 1$  2

Question 2 (12 marks)

(a) Differentiate with respect to  $x$

(i)  $y = \tan^3(5x + 4)$  2

(ii)  $y = \ln\left(\frac{2x+3}{3x+4}\right)$  2

(iii)  $y = \cos(e^{1-5x})$  2

(b) 30 girls, including Miss Australia, enter a Miss World Competition. The first six places are announced.

(i) How many different announcements are possible? 1

(ii) How many different announcements are possible if Miss Australia is assured a place in the first six? 2

(c) If  $f(x) = \tan^{-1}(2x)$  evaluate:

(i)  $f\left(\frac{1}{2}\right)$  1

(ii)  $f'\left(\frac{1}{2}\right)$  2

End of Section

Section B (Use a SEPARATE writing booklet)

Marks

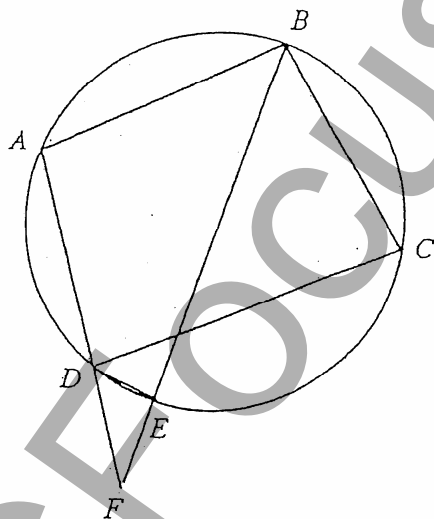
Question 3 (12 marks)

- (a) (i) State the natural domain and the corresponding range of  $y = 3 \cos^{-1}(x - 2)$  2
- (ii) Hence, or otherwise sketch  $y = 3 \cos^{-1}(x - 2)$  1
- (b) Find  $\int x\sqrt{16 + x^2} dx$  using the substitution  $u = 16 + x^2$  2
- (c) Find the general solution of  $\sin 2\theta = \sqrt{3} \cos 2\theta$  2
- (d) The roots of the equation  $4x^3 + 6x^2 + c = 0$ , where  $c$  is a non-zero constant, are  $\alpha$ ,  $\beta$ , and  $\alpha\beta$ . 5
- (i) Show that  $\alpha\beta \neq 0$ .
- (ii) Show that  $\alpha\beta + \alpha^2\beta + \alpha\beta^2 = 0$  and deduce the value of  $\alpha + \beta$ .
- (iii) Show that  $\alpha\beta = -\frac{1}{2}$ .

Question 4 (12 marks)

- (a) If  $\tan \theta = 2$  and  $0 < \theta < \frac{\pi}{2}$  evaluate  $\sin\left(\theta + \frac{\pi}{4}\right)$ . 3

- (b) In the diagram ABCD is a cyclic quadrilateral. The bisector of  $\angle ABC$  cuts the circle at E, and meets AD produced at F.

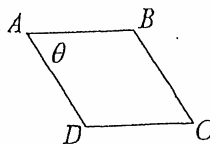
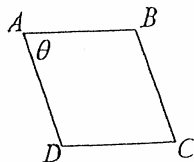
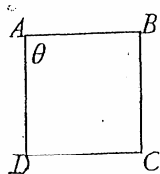


- (i) Copy the diagram showing the above information
- (ii) Give a reason why  $\angle CDE = \angle CBE$
- (iii) Show that DE bisects  $\angle CDF$

1

3

(c)



A square ABCD of side 1 unit is gradually 'pushed over' to become a rhombus. The angle at A ( $\theta$ ) decreases at a constant rate of  $0.1$  radians per second.

- (i) At what rate is the area of the rhombus ABCD decreasing when  $\theta = \frac{\pi}{6}$  ?

2

- (ii) At what rate is the shorter diagonal of the rhombus ABCD decreasing when  $\theta = \frac{\pi}{3}$  ?

3

End of Section

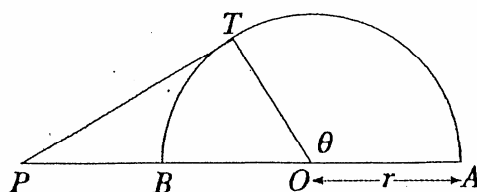
Section C (Use a SEPARATE writing booklet)

Marks

Question 5 (12 marks)

- (a) Two boys decide to settle an argument by taking turns to toss a die. The first person to throw a six wins.
- (i) What is the probability that the first person wins on his second throw? 1
- (ii) What is the probability that the first person will win the argument? 2
- (b)  $P(2at, at^2)$ ,  $t > 0$  is a point on the parabola  $x^2 = 4ay$ .  
The normal to the parabola at P cuts the  $x$  axis at X and the  $y$  axis at Y.
- (i) Show that the normal at P has equation  $x + ty - 2at - at^3 = 0$  2
- (ii) Find the co-ordinates of X and Y 1
- (iii) Find the value of  $t$  such that P is the midpoint of XY 2

(c)



The point  $T$  lies on the circumference of a semicircle, radius  $r$  and diameter  $AB$ , as shown. The point  $P$  lies on  $AB$  produced and  $PT$  is the tangent at  $T$ .

4

The arc  $AT$  subtends an angle of  $\theta$  at the centre,  $O$ , and the area of  $\triangle OPT$  is equal to that of the sector  $AOT$ .

(i) Show that  $\theta + \tan \theta = 0$ .

(ii) Taking 2 as an approximation to  $\theta$ , use Newton's method once to find a better approximation to two decimal places.



Question 6 (12 marks)

- (a) A particle is oscillating in simple harmonic motion such that its displacement  $x$  metres from a given origin  $O$  satisfies the equation  $\frac{d^2x}{dt^2} = -4x$  where  $t$  is the time in seconds
- (i) Show that  $x = \alpha \cos(2t + \beta)$  is a possible equation of motion for this particle, where  $\alpha$  and  $\beta$  are constants 2
- (ii) The particle is observed initially to have a velocity of 2 metres per second and a displacement from the origin of 4 metres. Find the amplitude of the oscillation. 2
- (iii) Determine the maximum velocity of the particle 2
- (b) Prove by Mathematical Induction that 3
- $$\sum_{r=1}^n r^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4}n^2(n+1)^2$$
- (c) Consider the function  $f(x) = \frac{x}{\sqrt{1-x^2}}$
- (i) Find the domain of  $f(x)$  1
- (ii) Find  $f^{-1}(x)$ , the inverse function of  $f(x)$  2

End of Section

**Section D** (Use a SEPARATE writing booklet)

Marks

Question 7 (12 marks)

- (a) A projectile fired with velocity  $V$  and at an angle of  $45^\circ$  to the horizontal, just clears the tops of two vertical posts of height  $8a^2$ , and the posts are  $12a^2$  apart. There is no air resistance, and the acceleration due to gravity is  $g$ .
- (i) If the projectile is at a point  $P(x, y)$  at time  $t$ ,  
Derive expressions for  $x$  and  $y$  in terms of  $t$ . 2
- (ii) Hence, show that the equation of the path of the projectile  
is  $y = x - \frac{gx^2}{V^2}$  2
- (iii) Using the information in (ii) show that the range of the  
projectile is  $\frac{V^2}{g}$  2
- (iv) If the first post is  $b$  units from the origin, show that 2
- (α)  $\frac{V^2}{g} = 2b + 12a^2$
- (β)  $8a^2 = b - \frac{gb^2}{V^2}$
- (v) Hence or otherwise prove that  $V = 6a\sqrt{g}$  4

End of paper