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Student Number



CATHOLIC SECONDARY SCHOOLS  
ASSOCIATION OF NEW SOUTH WALES

**2004**

**TRIAL HIGHER SCHOOL CERTIFICATE  
EXAMINATION**

# Mathematics

## Extension 1

Afternoon Session  
Tuesday 10 August 2004

### General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question

**Total marks – 84**

- Attempt Questions 1-7
- All questions are of equal value

### Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

## Question 1

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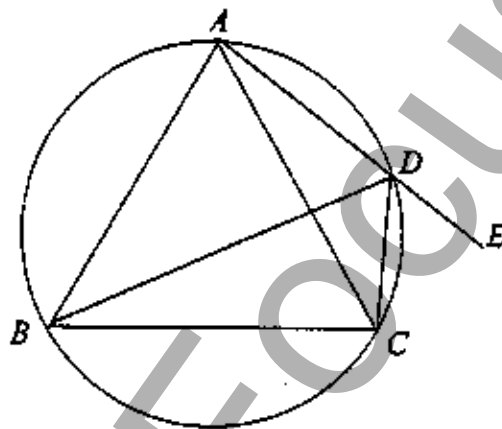
✓ (a) Find  $\frac{d}{dx}(1+x^2)\tan^{-1}x$ . 2

(b) The polynomial  $P(x)$  is given by  $P(x) = x^3 + ax + 1$  for some real number  $a$ .  
The remainder when  $P(x)$  is divided by  $(x-1)$  is equal to the remainder when  $P(x)$  is divided by  $(x-2)$ . Find the value of  $a$ . 2

(c) (i) The line  $y = mx$  makes an angle of  $45^\circ$  with the line  $y = 2x$ . Show that  $|m-2| = |1+2m|$ . 2

(ii) Hence find the equations of the lines  $y = mx$  which make an angle of  $45^\circ$  with the line  $y = 2x$ . 2

(d)



$ABC$  is a triangle in which  $BC = AC$ .  $D$  is a point on the minor arc  $AC$  of the circle passing through  $A$ ,  $B$  and  $C$ .  $AD$  is produced to  $E$ .

(i) Copy the diagram.

(ii) Give a reason why  $\angle CDE = \angle ABC$ . 1

(iii) Hence show that  $DC$  bisects  $\angle BDE$ . 3

## Question 2

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- (a)  $A(-5,6)$  and  $B(1,3)$  are two points. Find the coordinates of the point  $P$  which divides the interval  $AB$  externally in the ratio  $5:2$ . 2

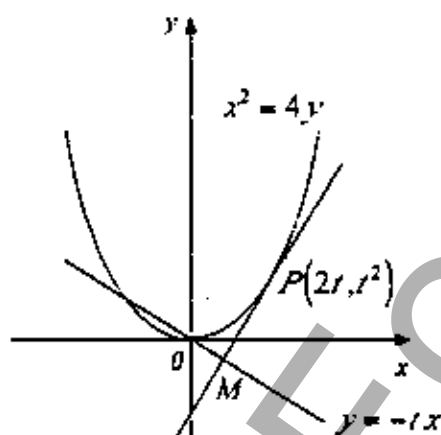
- (b) The equation  $2x^3 + 2x^2 + 4x + 1 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ . Find the value of  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ . 2

- (c) Consider the geometric series  $\sin 2x + \sin 2x \cos 2x + \sin 2x \cos^2 2x + \dots$  for  $0 < x < \frac{\pi}{2}$ .

- (i) Show that the limiting sum  $S$  of the series exists. 2

- (ii) Show that  $S = \cot x$ . 2

- (d)



$P(2t, t^2)$  is a point on the parabola  $x^2 = 4y$ . The tangent to the parabola at  $P$  and the line  $y = -tx$  intersect at the point  $M$ .

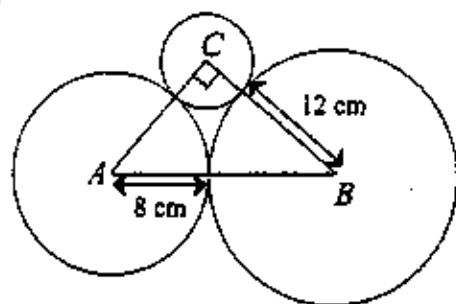
- (i) Show that the tangent to the parabola at  $P$  has gradient  $t$  and equation  $x - y - t^2 = 0$ . 2

- (ii) Find the Cartesian equation of the locus of  $M$  as  $t$  varies. 2

## Question 3

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(a)



Three circles with centres  $A$ ,  $B$  and  $C$  touch externally in pairs with  $\angle BCA = 90^\circ$ , as shown in the diagram. The circles with centres  $A$  and  $B$  have radii 8 cm and 12 cm respectively.

- (i) If the circle with centre  $C$  has radius  $x$  cm, show that  $x^2 + 20x - 96 = 0$ . 2
- (ii) Hence find the radius of the circle with centre  $C$ . 2
- (b) (i) Show that  $\frac{u}{u+1} = 1 - \frac{1}{u+1}$ . 1
- (ii) Hence find  $\int \frac{1}{1+\sqrt{x}} dx$  using the substitution  $x = u^2$ ,  $u \geq 0$ . 3
- (c) Use Mathematical Induction to show that  $5^n > 4^n + 3^n$  for all integers  $n \geq 3$ . 4

## Question 4

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- (a) Find the term independent of  $x$  in the binomial expansion of  $\left(x - \frac{2}{x^2}\right)^{15}$ . 3
- (b) After  $t$  years,  $t \geq 0$ , the number  $N$  of individuals in a population is given by  $N = A + Be^{-kt}$  for some constants  $A > 0$  and  $B > 0$ . The initial population size is 500 individuals and the limiting population size is 100 individuals.
- (i) Find the values of  $A$  and  $B$ . 2
- (ii) Find the time taken for the population size to fall within 10 of its limiting value, giving the answer correct to the nearest month. 2
- (c) Consider the function  $f(x) = x - \cos x$ .
- (i) Show that the equation  $f(x) = 0$  has a root  $\alpha$  such that  $0 < \alpha < 1$ . 2
- (ii) Use one application of Newton's Method with an initial approximation of 0.7 to approximate  $\alpha$ , giving the answer correct to 2 decimal places. 3

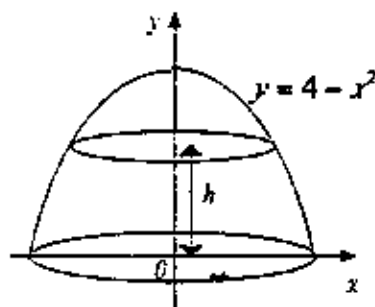
## Question 5

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(a) In a certain street, 40% of the households have at least 2 cars. If 4 households are chosen at random, find the probability that

- (i) exactly 3 of these households have at least 2 cars. 1  
 (ii) at most 3 of these households have at least 2 cars. 2

(b)



A mould for a container is made by rotating the part of the curve  $y = 4 - x^2$  which lies in the first quadrant through one complete revolution about the  $y$  axis. After sealing the base of the container, water is poured through a hole in the top. When the depth of water in the container is  $h$  cm, the depth is changing at a rate  $\frac{10}{\pi(4-h)} \text{ cms}^{-1}$ .

(i) Show that when the depth is  $h$  cm, the surface area  $S \text{ cm}^2$  of the water is given by  $S = \pi(4 - h)$ . 1

(ii) Find the rate at which the surface area of the water is changing when the depth of the water is 2 cm. 3

(c) Consider the function  $f(x) = e^x - x$ .

(i) Show that the curve  $y = f(x)$  is concave up for all values of  $x$ . 1

(ii) Find the coordinates and nature of the stationary point on the curve  $y = f(x)$ . 2

(iii) Hence show that  $e^x \geq x + 1$  for all values of  $x$ . 2

## Question 6

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(a) Consider the function  $f(x) = \cos^{-1}(x - 1)$ .

(i) Find the domain of the function. 1

(ii) Sketch the graph of the curve  $y = f(x)$  showing clearly the coordinates of the endpoints. 2

(iii) The region in the first quadrant bounded by the curve  $y = f(x)$  and the coordinate axes is rotated through one complete revolution about the  $y$  axis. Find the exact value of the volume of the solid of revolution. 3

(b) A particle moving in a straight line is performing Simple Harmonic Motion. At time  $t$  seconds it has displacement  $x$  metres to the right of a fixed point  $O$  on the line, where  $x = 4 \cos^2 t - 2 \sin^2 t$ .

- (i) Show that  $x = 1 + 3 \cos 2t$ . Hence express the acceleration  $\ddot{x} \text{ ms}^{-2}$  of the particle in the form  $\ddot{x} = -n^2(x - b)$ , where the values of the constants  $n$  and  $b$  are to be determined. 2
- (ii) Find the set of possible values of  $x$  and the period of the motion. 2
- (iii) Find the distance travelled and the time taken (to the nearest tenth of a second) for the particle to first pass through  $O$ . 2

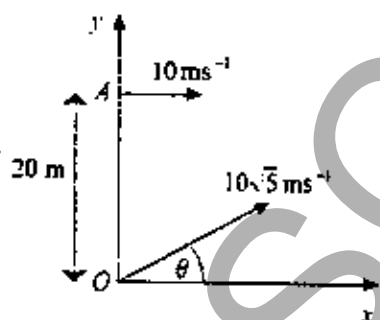
## Question 7

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(a) A particle is moving in a straight line. At time  $t$  seconds it has displacement  $x$  metres to the right of a fixed point  $O$  on the line and velocity  $v \text{ ms}^{-1}$  given by  $v = \sin x \cos x$ . The particle starts  $\frac{\pi}{4}$  metres to the right of  $O$ .

- (i) Show that  $\frac{d}{dx} \ln(\tan x) = \frac{1}{\sin x \cos x}$ . 1
- (ii) Hence show that the displacement of the particle is given by  $x = \tan^{-1}(e^t)$ . 3
- (iii) Find the limiting position of the particle and sketch the graph of  $x$  against  $t$ . 2

(b)



OA is a vertical building of height 20 metres. A particle is projected horizontally from A with speed  $10 \text{ ms}^{-1}$ . At the same instant, a second particle is projected from O with speed  $10\sqrt{5} \text{ ms}^{-1}$  at an angle  $\theta$  above the horizontal. The two particles travel in the same plane of motion. Take  $g = 10 \text{ ms}^{-2}$ .

- (i) Write down expressions for the horizontal and vertical displacements relative to  $O$  of each particle after time  $t$  seconds. 2
- (ii) Show that if the two particles collide, then they do so after 1 second. 2
- (iii) Show that if the two particles collide, when they do so their paths of motion are perpendicular to each other. 2